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Diurnal and semi-diurnal oscillations of the lower stratosphere

by
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Summary

It is demonstrated that diurnal and semi-diurnal variations of wind occur in the lower stratosphere. These oscillations appear to be associated with the solar tides. The stratospheric semi-diurnal oscillation of wind is in phase with, and of comparable magnitude to, the surface semi-diurnal oscillation of wind. In the stratosphere, the diurnal oscillation is found to be of similar magnitude to the semi-diurnal oscillation.

1. Introduction

Whilst methods of representing the 100 mb wind field were under investigation, a diurnal variation was detected in the winds observed at the 100 mb level. A study of the periodic variations of wind in the lower stratosphere has been made in order to establish their significance, form and magnitude.

2. Reduction of the data

Occasions in 1950-51 were selected when winds were reported at 100 mb from a British upper air station at times t hr and t + 6 hr on the same day. Four sets of data were extracted corresponding to the four six-hour intervals 0900 G.M.T. - 0300 G.M.T., 1500 G.M.T. - 0900 G.M.T., 2100 G.M.T. - 1500 G.M.T. and 0300 G.M.T. - 2100 G.M.T. The observed winds were resolved into south and west components and two-year monthly mean components of the wind change over the 6 hr interval were computed for each period. These are reproduced in Table I.

No very critical statistical test is required to show that at least some of the annual means given in Table I differ significantly from zero e.g. for the 0900 G.M.T. - 0300 G.M.T. west component variation, all the monthly means have the same sign. If the monthly means themselves belonged to a population whose mean was zero, the odds against their having the same sign would be $(2^{11}-1) : 1$, i.e. 2047: 1. The magnitudes of the annual mean six-hour variations are less than the standard error (3 or 4 kt) of the individual wind observations and are also less than the six-hour synoptic variations. To obtain some indication of the general level of accuracy of the individual monthly means, an estimate has been made of the standard errors of 1500 - 0900 G.M.T. variations for four months. The individual values of the 1500 G.M.T. - 0900 G.M.T. variations do not form an independent set because, although the serial correlation for individual stations is negligible, a small but significant correlation does exist between simultaneous values at different stations. If the data were composed of simultaneous values

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from K stations and the average correlation coefficient between pairs of stations was $\bar{\ell}$, it can be shown that the standard deviation of the mean of n observations is given by

$$\sigma_M = \sigma\sqrt{\frac{1+(K-1)\bar{\ell}}{n}} \quad .$$

This replaces the usual relation for independent values of $\sigma_M = \sigma\sqrt{\frac{1}{n}}$. Using the former relation with appropriate values of K and $\bar{\ell}$ the standard errors of the monthly means in June, July, December and January respectively were estimated to be, for the south component, 0.43 kt, 0.53 kt, 1.00 kt and 0.70 kt; and for the west component, 0.39 kt, 0.42 kt, 0.65 kt and 0.54 kt. Evidently, a number of the monthly means themselves differ significantly from zero.

If a_1, a_2, a_3 and a_4 are the four true six-hour variations of wind then, neglecting the seasonal variation of wind, which is small in comparison with the diurnal changes, we can put $a_1 + a_2 + a_3 + a_4 = 0$. This relationship is not obeyed exactly by the variations given in Table I because, on many days, winds are lacking at one or more of the hours of observation. For these days, only one or two of the six-hour variations were obtained. We can thus obtain from Table I two estimates of each of the a_i; one is given directly by a_i, and the other is given by $-(a_j + a_k + a_l)$. The means of the two estimates are given as the "derived" monthly mean six-hour variations in Table II. These derived variations form the basis of the analysis which follows. They are illustrated graphically in Figure 1. Figure 2 shows the implied twelve-hour variations for each wind component.

Inspection of Figures 1 and 2 suggests that any seasonal variations which may occur in the diurnal changes are definitely smaller than the annual means. To test this impression, averages were calculated for the winter (November to March) and summer half years. These are given in Table III. "Student's" t-test applied to this data showed that the differences between the winter and summer means were in no case significant, even at the 10% level.

A test was also made to determine whether any of the differences between the monthly and annual mean six-hour variations was so great as to merit special investigation. The variances of the monthly means about the annual means averaged 0.26 kt for the west components and 0.46 kt for the south components. None of the west component monthly means differs significantly from the annual mean. For the south component, the 0900 G.M.T. - 0300 G.M.T. January monthly mean differs from the appropriate annual mean by an amount, 2.16 kt, equal to 3.19 times the standard deviation. The odds against such a difference occurring by chance on any specified occasion are a little under 999:1 but the odds against such a difference occurring by chance in a sample of 48 values are less than 957:43. The deviation of the January value from the annual mean is therefore of doubtful significance.

Since the data show no clear cut seasonal variation further study has been confined to the annual mean six and twelve-hour changes.

Soon after the first results of this work became known, H.W.L. Absalom generously made available for study by the writer, some further, very valuable, unpublished data. These data consist of six-hourly variations of wind at 150 mb calculated from British wind observations made during the years 1944-47, when upper winds were measured at the hours 0600, 1200, 1800 and 2400 G.M.T. These additional data are contained in Table IV.

3. Analysis of the oscillations

The sum of the four annual mean six-hourly variations should be zero. In order that the annual means given in Table II should satisfy this condition, and so to facilitate further analysis, they have been adjusted slightly. The south

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component variations have been increased by 0.01 kt and the west component variations by 0.12 kt. The appropriate deviations, corresponding to the adjusted six-hour variations, of the winds at each hour of observation relative to the overall mean wind are then readily calculated. The difference between the wind at each synoptic hour and the mean wind is given in Table V which, in effect, combines the results given in Table II with those given in Table IV.

The next step was to break down the oscillations represented by the data in Table V by harmonic analysis. For each wind component it is possible from this data to obtain four harmonics. These are given by the equations:

$$100\ S = -54 \cos(\theta + 32^\circ 14') + 67 \cos(2\theta + 44^\circ 24') - 6 \cos(3\theta + 80^\circ 32') + 0.25 \cos 4\theta ; \quad (1)$$

$$100\ W = +51 \cos(\theta + 84^\circ 24') + 51 \cos(2\theta - 29^\circ 3') - 7 \cos(3\theta - 26^\circ 34') + 0.0 \cos 4\theta ; \quad (2)$$

where S and W are the south wind and west wind components in kt, and θ varies from 0 to 2π as time varies through 24 hours from 0300 G.M.T.

Equations (1) and (2) reveal that the oscillation can be accounted for largely by the first two harmonics, i.e. by two component oscillations, one having a period of twenty four hours and the other having a period of twelve hours. A test has been made to determine to what extent the individual harmonics could account for the observed variations. Table VI gives the values of "Student's" t obtained for each of the six-hour variations given in Table II, in order to examine the following possibilities, viz., that the twelve monthly means are drawn from a population whose mean is (a) zero, (b) given by the first harmonic, (c) given by the second harmonic, (d) given by the first and second harmonics combined or (e) given by the sum of the first three harmonics. For each of these tests values of t, corresponding to probabilities of 0.1%, 1%, 5% and 10% that the sample tested is drawn from a population with the postulated mean value, are 4.44, 3.11, 2.20 and 1.80 respectively. From Table VI we deduce that the observed system of variations cannot be accounted for satisfactorily on the assumptions (a) that there is no real variation, (b) that they consist of a diurnal oscillation alone or (c) that they consist entirely of a semi-diurnal oscillation alone. The variations can be explained satisfactorily by a combination of the diurnal and semi-diurnal oscillations: it is not necessary to invoke harmonics of higher order than the second to account for the present data.

4. The diurnal and semi-diurnal oscillations

The diurnal oscillations of the south wind and west wind components are defined by the first terms on the right hand sides of equations (1) and (2). These have been combined in Figure 3, which is a vector diagram giving, for any time, the resultant wind velocity due to the diurnal oscillations. The semi-diurnal variation of wind, given by the second terms on the right hand sides of equations (1) and (2), is represented in a similar fashion in Figure 4.

It was demonstrated above that we can account very largely for the observed six-hour variations by combining the diurnal and semi-diurnal oscillations. Figure 5 shows the result of superimposing one oscillation on the other and also gives, for comparison, the observed variations.

The elliptical paths which would be followed by particles moving with the wind systems defined in Figures 3 and 4 are shown in Figures 6 (a) and 6 (b). Figure 6 (c) depicts the kidney-shaped path followed by particles undergoing the combined oscillation.

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5. Physical nature of the oscillations

In the preceding sections it has been established that the measured changes of wind in the lower stratosphere contain small periodic components. Possible explanations of the periodic variations are: (i) that they are spurious and are due to some systematic error of the wind measurements or (ii) that they are real and are associated with the tidal oscillations which are known to occur in the atmosphere.

It has not been possible to conceive of any direct instrumental effect which could account for the observed variations. The 1944-47 winds were measured for the most part by radio direction-finding techniques, whereas the 1950-51 winds were measured by the use of G.L. III radar equipment. It is difficult to envisage any inherent periodic instrumental effects which could be common to both techniques.

Some diurnal effect could arise as follows. There is an apparent diurnal variation in the measured height of the 100 mb surface. The apparent diurnal 1500 G.M.T. - 0300 G.M.T. change of contour height ranges from about 140 ft at the winter solstice to 380 ft at the summer solstice, for stations in the British Isles.

It is believed that this apparent variation in height is largely false, and that it results from the effect of solar radiation on the radio-sonde itself. It is likely that the 1500 G.M.T. "100 mb" winds refer to a level which is in fact from 140 ft to 380 ft higher than the true 100 mb surface. Consequently the wind observations will contain a false diurnal variation, the magnitude of which depends on the shear of wind at 100 mb. An examination of the shear of the mean wind in the lower stratosphere showed that the shear in the south component is so small as to have a completely negligible effect at all seasons. At the most favourable time of the year (June and July), the shear could give rise to an apparent 1500 G.M.T. - 0300 G.M.T. variation in the west component of about 0.4 kt, but from December to May the effect of shear in the west component is again negligible. Inspection of Figure 3 shows that the annual mean 1500 G.M.T. - 0300 G.M.T. diurnal change of west wind component is, in fact, quite small. The greatest twelve-hour change of west wind takes place between about 0900 G.M.T. and 2100 G.M.T., a period when there is little apparent variation of contour height. It is clear that for the annual means, we can discount the effect of the apparent diurnal variation of height of the 100 mb surface as being small in comparison with the observed diurnal variation of wind.

Atmospheric tides have been extensively studied in terms of the changes of pressure which occur at the earth's surface. The most important solar tides have periods of twelve and twenty four hours (1). Of these two, the semi-diurnal oscillation has attracted greater attention, partly because, at the earth's surface, it has the greater amplitude and shows greater uniformity than the diurnal oscillation. Simpson (2) has given the following expression for the observed semi-diurnal pressure variation;

$$p = 0.937 \cos^3 \theta \sin [2 (\omega t + \lambda) + 154^\circ] \qquad (3)$$

where p is pressure in mm of mercury, θ is latitude, ω is the earth's angular velocity, t is Greenwich mean time and λ is longitude east of Greenwich. Bartels (3) has derived theoretically, the following expressions for the low-level wind components associated with the pressure variation given by equation (3):

$$U = - 54.6 \sin \theta \cos [2 (\omega t + \lambda) + 154^\circ]$$

$$V = - (21.8 + 32.8 \sin^2 \theta) \sin [2 (\omega t + \lambda) + 154^\circ] \qquad (4)$$

where U is the south component and V is the west component of wind in cm/sec.

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It follows from expressions (3) and (4) that the pressure and west wind component oscillations are in antiphase at the earth's surface, maximum west wind component occurring at about 0400 and 1600 local time. In the northern hemisphere, the pressure and south wind component oscillations are in quadrature, maximum south wind component occurring at about 0100 and 1300 local time. Reference to Figure 4 shows that the semi-diurnal oscillation in the lower stratosphere is approximately in phase with the surface oscillation. This relationship is to be expected from modern tidal thoery (1).

The amplitudes of the surface tidal wind components as given by expression (4) are in latitude 55°N, 0.87 kt for the south component and 0.85 kt for the west component. These may be compared with the lower stratosphere semi-diurnal amplitudes of 0.67 kt for the south component and 0.51 kt for the west component. It appears that there is no great variation in the amplitude of the tidal winds, at least up to the 100 mb (16 km) level. It is interesting to note however, that recent studies, by Elford and Robertson (4) and others, of the drift of meteor trails, indicate that the semi-diurnal variations in the wind components have magnitudes of the order of 60 kt in the 80 km to 100 km layer.

As previously mentioned, the surface diurnal pressure changes have not been studied to the same extent as the semi-diurnal pressure changes. The present study indicates that the diurnal and semi-diurnal oscillations are of comparable magnitude in the lower stratosphere. Elford and Robertson's data (4) show that the two effects are also of equal importance in the 80 km - 100 km layer.

There appears to be little doubt that the periodic variations of wind discussed in this paper are largely associated with the solar tides. Wind observations made at 100 mb are particularly suitable for study in connection with the tidal oscillations since, at least between latitudes 30°N and 60°N, the synoptic variations of wind are considerably smaller at that level than they are in the upper troposphere. The lower stratosphere is also free from the smaller scale effects which would complicate studies of periodic wind variations occurring in the lower troposphere.

References

(1) Wilkes, M.V. 1949 Oscillations of the Earth's Atmosphere. Camb. Univ. Press.

(2) Simpson, G.C. 1918 Quart. J. Roy. Met.Soc. 44, p.20.

(3) Bartels, J. 1928 Handbuch der Experimentalphysik. 25, p.163.

(4) Elford, W.G. and Robertson, D.S. 1953 J. Atmosph. Terr. Phys. 4, p.271.

TABLE I. THE OBSERVED MONTHLY MEAN SIX-HOUR CHANGES OF WIND AT 100 mb OVER THE BRITISH ISLES FOR 1950-51

Component (kt)	Interval G.M.T.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Annual Mean
West	0900-0300	-0.85	-1.20	-1.25	-1.17	-1.42	-1.42	-2.40	-0.91	-0.73	-0.93	-1.77	-0.36	-1.23
	No. of cases	111	90	106	110	150	151	143	120	121	151	87	90	1430
	1500-0900	0.48	0.85	1.96	1.17	1.51	1.50	2.30	1.93	1.52	1.90	2.00	0.02	1.50
	No. of cases	134	119	169	166	212	200	185	175	146	180	104	107	1897
	2100-1500	0.16	-0.42	0.04	-0.56	0.50	-0.41	-1.09	-0.11	0.07	-0.92	-0.67	-0.44	-0.32
	No. of cases	123	97	121	103	134	141	129	134	120	165	92	120	1479
	0300-2100	0.58	0.13	0.32	0.47	0.15	0.43	1.14	0.46	0.30	0.51	1.17	0.62	0.52
	No. of cases	128	77	100	92	109	141	120	107	116	165	84	101	1340
South	0900-0300	-1.58	-0.81	0.07	-0.10	0.26	-0.07	-0.22	-0.03	-0.90	0.25	-0.33	0.10	-0.24
	No. of cases	111	90	106	110	150	151	143	120	121	151	87	90	1430
	1500-0900	2.38	2.03	1.30	1.42	0.76	0.62	1.64	0.88	2.43	0.83	0.36	-0.16	1.23
	No. of cases	134	119	169	166	212	200	185	175	146	180	104	107	1897
	2100-1500	-0.94	-3.48	-2.04	-1.08	-1.70	-1.65	-2.01	-1.96	-1.99	-1.16	-1.88	-0.23	-1.64
	No. of cases	123	97	121	103	134	141	129	134	120	165	92	120	1479
	0300-2100	1.95	1.33	0.85	1.09	0.32	0.78	0.37	-0.73	0.59	0.71	1.21	0.14	0.72
	No. of cases	128	77	100	92	109	141	120	107	116	165	84	101	1340

/TABLE II.

TABLE II. THE "DERIVED" MONTHLY MEAN SIX-HOUR CHANGES OF WIND AT 100 mb OVER THE BRITISH ISLES FOR 1950-51.

Component (kt)	Interval G.M.T.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Annual Mean
West	0900-0300	-1.03	-0.88	-1.79	-1.12	-1.79	-1.47	-2.37	-1.59	-1.31	-1.21	-2.13	-0.28	-1.47
	1500-0900	0.29	1.17	1.43	1.21	1.14	1.45	2.33	1.24	0.94	1.62	1.63	0.10	1.27
	2100-1500	-0.03	-0.10	-0.49	-0.51	0.13	-0.46	-1.06	-0.79	-0.51	-1.20	-1.03	-0.36	-0.55
	0300-2100	0.39	0.45	-0.21	0.51	-0.22	-0.38	1.16	-0.22	-0.28	0.23	0.80	0.70	0.29
South	0900-0300	-2.43	-0.35	-0.01	-0.76	0.34	0.04	-0.11	0.89	-0.96	-0.06	-0.01	0.17	-0.27
	1500-0900	1.47	2.49	1.21	0.75	0.94	0.78	1.75	1.80	2.37	0.56	0.68	-0.09	1.19
	2100-1500	-1.84	-3.01	-2.13	-1.74	-1.50	-1.49	-1.90	-1.04	-2.05	-1.47	-1.56	-0.15	-1.67
	0300-2100	1.04	1.79	0.76	0.42	0.50	0.94	0.48	0.19	0.52	0.39	1.53	0.21	0.69

/TABLE III.

TABLE III. MEAN SIX-HOUR VARIATIONS OF 100 mb WIND FOR WINTER AND SUMMER HALF YEARS, 1950-51

Interval G.M.T.	West Component (kt)		South Component (kt)	
	Summer	Winter	Summer	Winter
0900 - 0300	-1.61	-1.22	-0.09	-0.45
1500 - 0900	1.39	1.04	1.40	1.05
2100 - 1500	-0.53	-0.53	-1.62	-1.69
0300 - 2100	0.09	0.39	0.51	0.95

/TABLE IV.

TABLE IV. THE OBSERVED MONTHLY MEAN SIX-HOUR CHANGES OF WIND AT 150 mb OVER THE BRITISH ISLES FOR 1944-47 * (DUE TO ABSALOM)

Component (kt)	Interval G.M.T.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Annual Mean
West	0600-0000	00	1.6	0.6	-1.6	0.5	-0.9	-0.5	-1.1	-2.5	0.0	-0.9	2.5	-0.2
	1200-0600	0.4	-0.8	-2.7	1.2	-0.9	0.1	1.3	-0.7	-0.4	-1.5	-0.5	-2.3	-0.6
	1800-1200	1.8	0.8	0.6	1.3	1.5	0.5	0.7	1.7	2.0	1.5	3.0	0.3	1.3
	0000-1800	1.7	-2.1	1.3	-1.7	-0.3	-1.5	-0.3	0.1	-1.1	-0.2	0.3	0.2	-0.3
South	0600-0000	-1.3	0.7	0.6	-0.5	1.0	0.7	-1.9	-1.3	-0.7	0.1	1.3	-2.3	-0.3
	1200-0600	1.7	0.3	0.8	2.2	1.5	1.7	2.5	1.8	0.2	0.6	3.8	3.6	1.7
	1800-1200	-0.2	-1.5	-1.3	-1.3	-1.5	-1.8	-1.4	-1.1	-0.1	-1.9	-2.2	-1.4	-1.3
	0000-1800	3.1	0.8	-1.1	0.5	0.6	-1.4	0.1	-1.0	1.1	-3.4	4.6	-0.1	0.3

* The 0000-1800 variations were calculated from 2 years data.

/TABLE V.

TABLE V. DEVIATION OF ANNUAL MEAN WIND COMPONENT FOR STATED HOURS FROM THE OVERALL MEANS

Time G.M.T.	0300	0600	0900	1200	1500	1800	2100	2400
South Component (kt)	0.01	-0.54	-0.25	1.03	0.95	-0.40	-0.71	-0.11
West Component (kt)	0.43	-0.05	-0.92	-0.70	0.46	0.55	0.03	0.20

/TABLE VI

TABLE VI. VALUES OF "STUDENT'S" t OBTAINED IN TESTS OF THE SIGNIFICANCE OF THE OBSERVED SYSTEM OF SIX-HOUR VARIATIONS

Component	Period G.M.T.	A	B	C	D	E
South	0900-0300	-1.13	-4.25	2.87	-0.25	-0.04
	1500-0900	5.59	4.82	1.23	0.45	0.18
	2100-1500	8.74	-4.79	-3.68	0.26	-0.05
	0300-2100	4.87	6.00	-1.53	-0.40	0.07
West	0900-0300	-8.29	-5.00	-3.00	0.29	-0.23
	1500-0900	7.12	4.41	1.82	-0.88	-1.06
	2100-1500	-4.42	-9.08	3.08	-1.58	-0.83
	0300-2100	1.60	4.67	-4.40	-1.33	-1.13

Column "A" gives values of t obtained for difference between observed variations and zero variation.
Column "B" gives values of t obtained for difference between observed variations and the first harmonic of the observed variation.
Column "C" gives values of t obtained for difference between observed variations and the second harmonic of the observed variation.
Column "D" gives values of t obtained for difference between observed variations and the first two harmonics of the observed variation.
Column "E" gives values of t obtained for difference between observed variations and the first three harmonics of the observed variation.

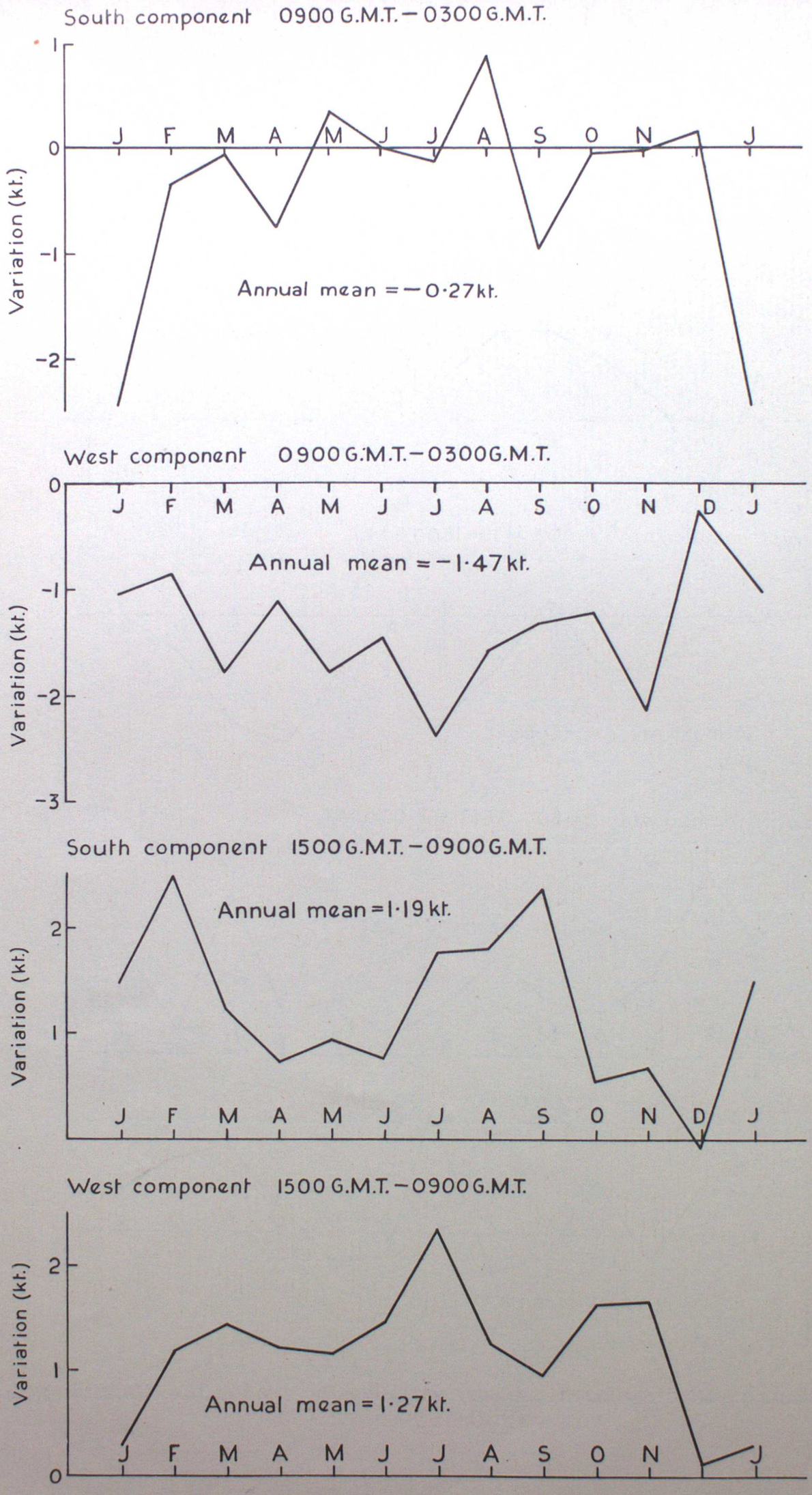

Fig. 1 Monthly mean six-hour variations of wind at the 100 mb. level 1950 – 1951

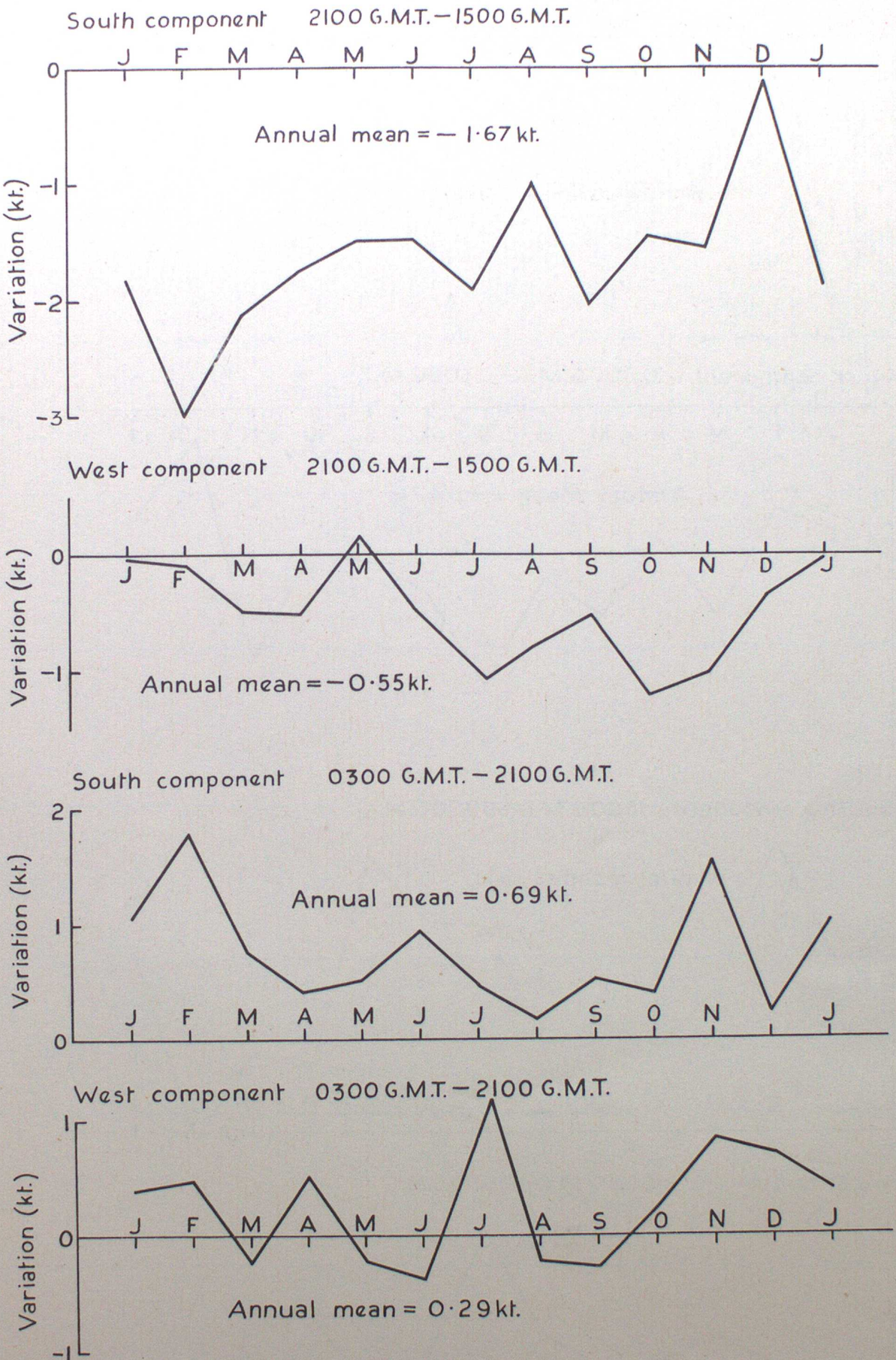

Fig.1 (cont'd.) Monthly mean six-hour variations of wind at the 100mb. level 1950–1951

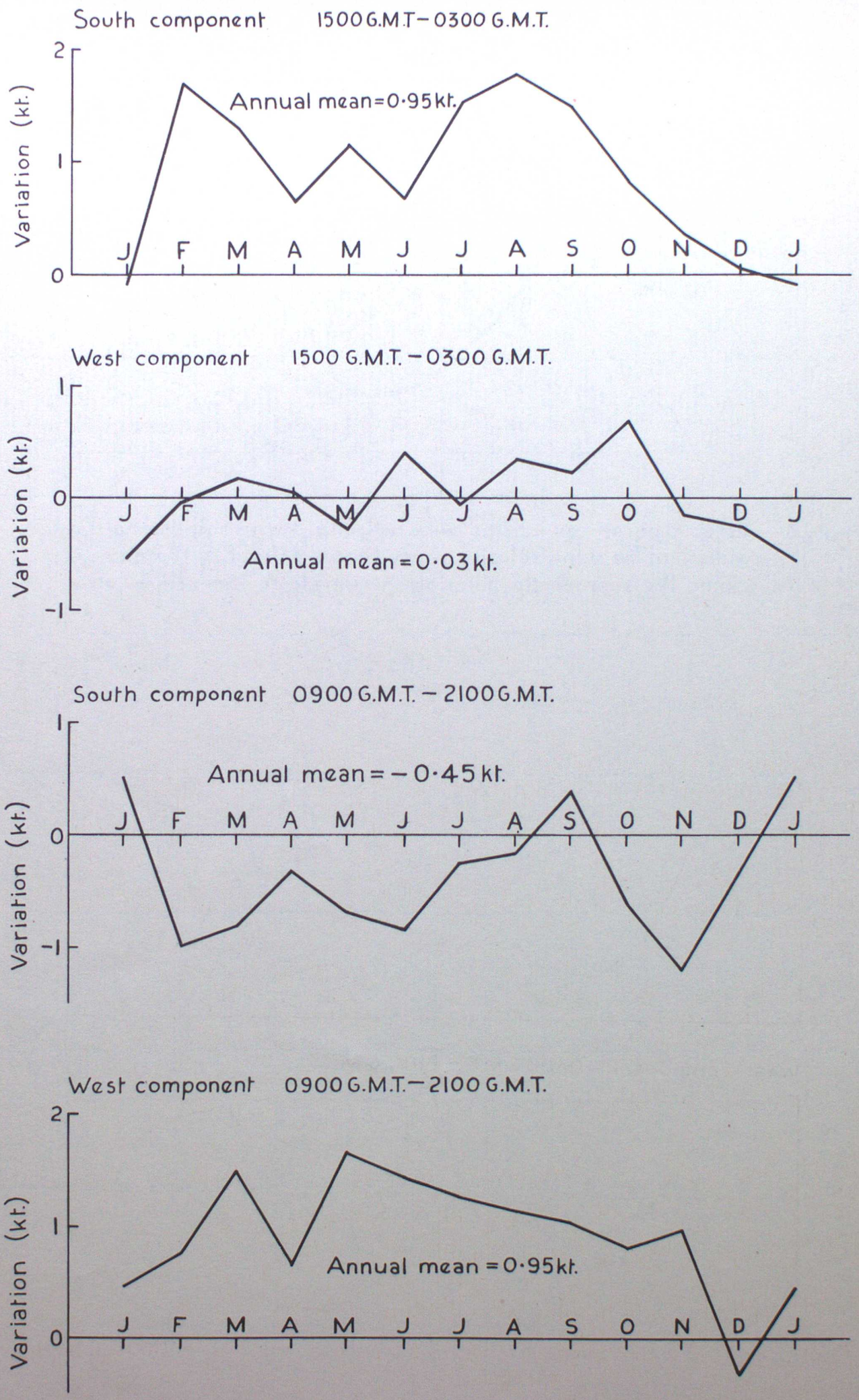

Fig. 2 Monthly mean twelve-hour variations of wind at the 100 mb. level 1950–1951

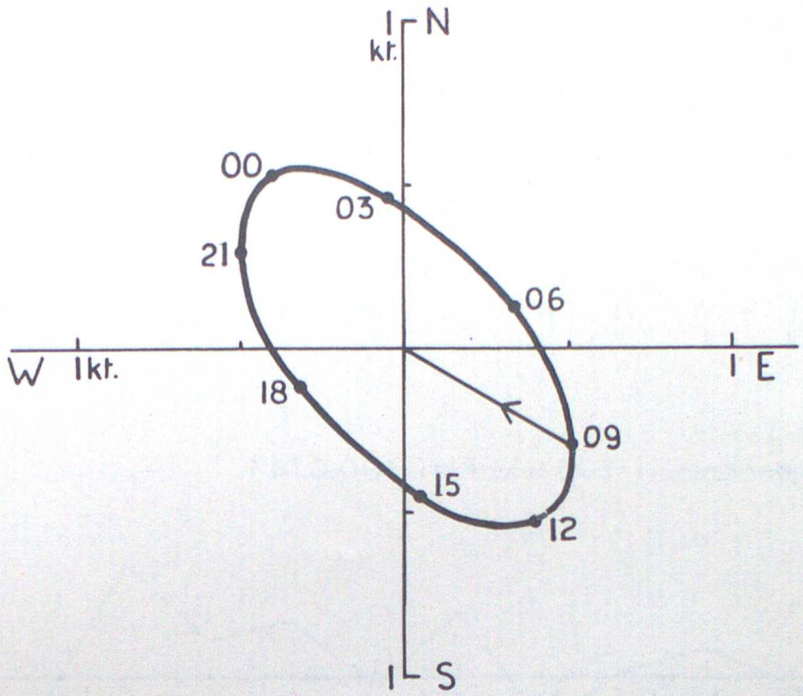

Fig. 3 Vector diagram giving the wind velocity due to the diurnal oscillation. The wind velocity at any time is defined by the line joining the appropriate point on the ellipse to the origin

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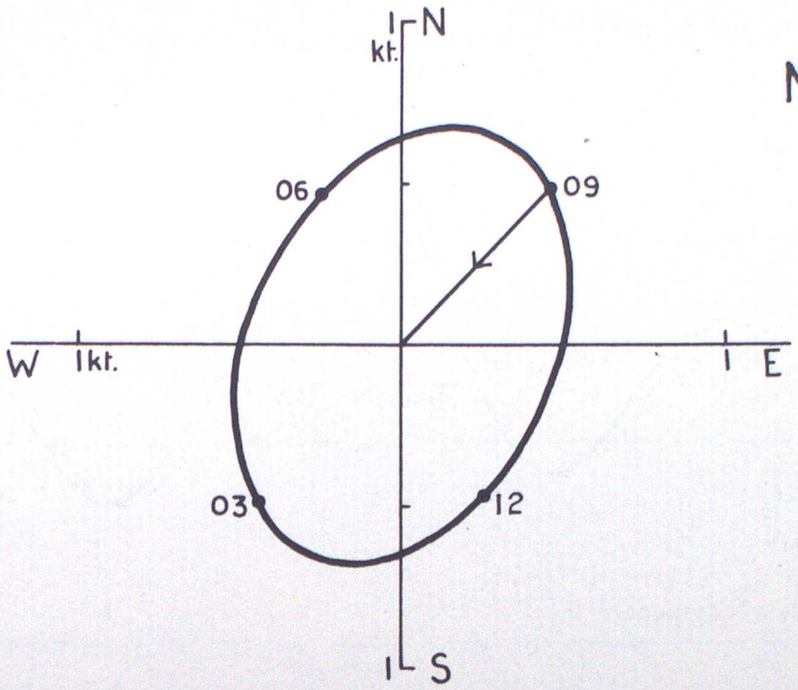
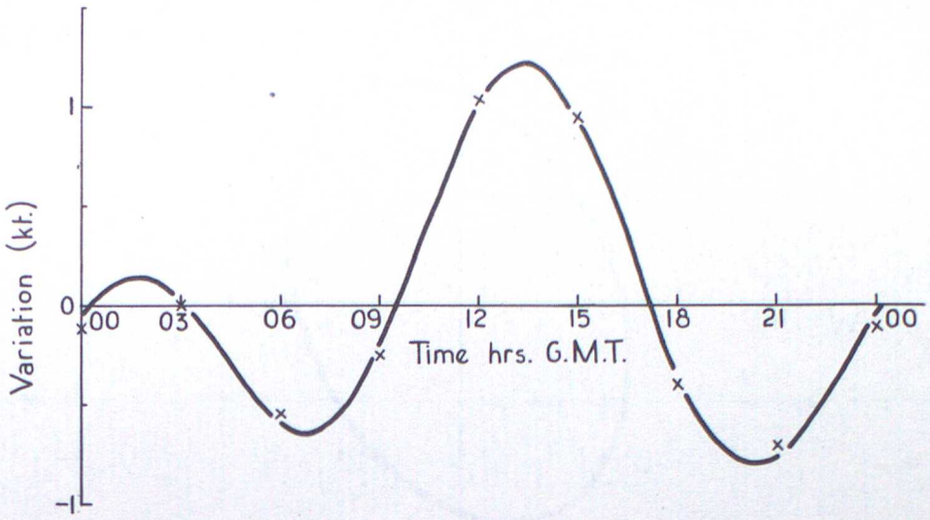
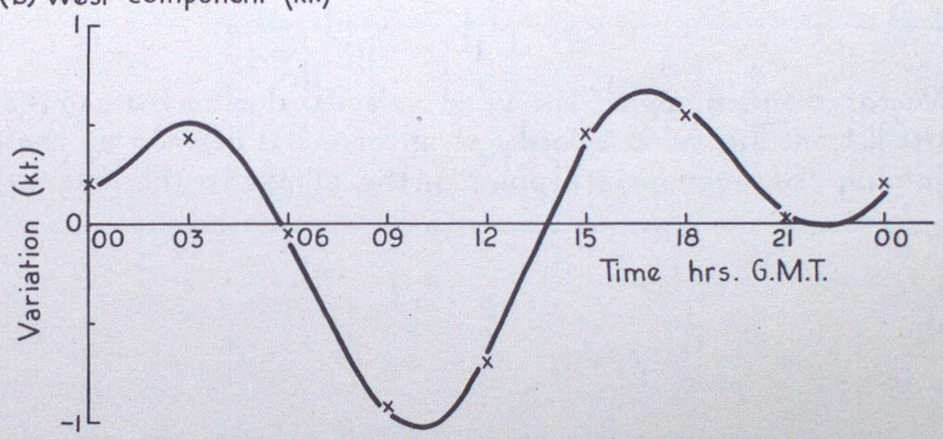


Fig. 4 Vector diagram giving the wind velocity due to the semi-diurnal oscillation. The wind velocity at any time is defined by the line joining the appropriate point on the ellipse to the origin

(a) South component (kt.)



(b) West component (kt.)



(c) Vector diagram giving the resultant wind velocity due to the combined diurnal and semi-diurnal oscillations. The wind velocity at any time is defined by the line joining the appropriate point on the curve to the origin

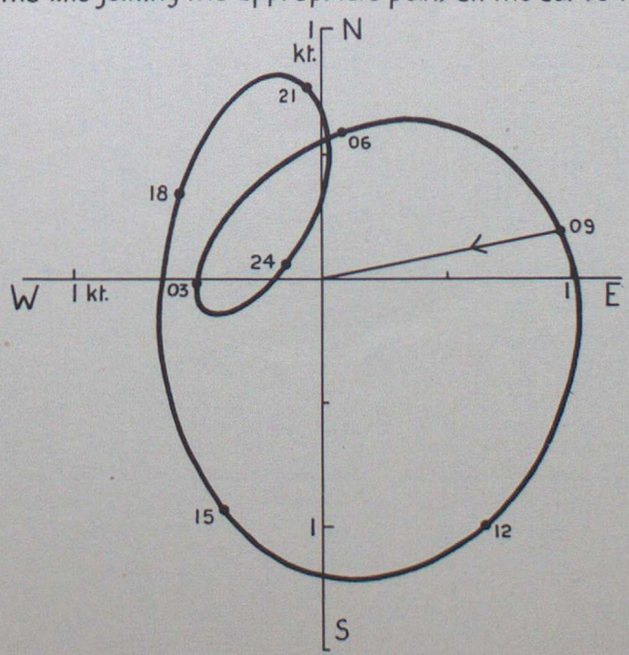
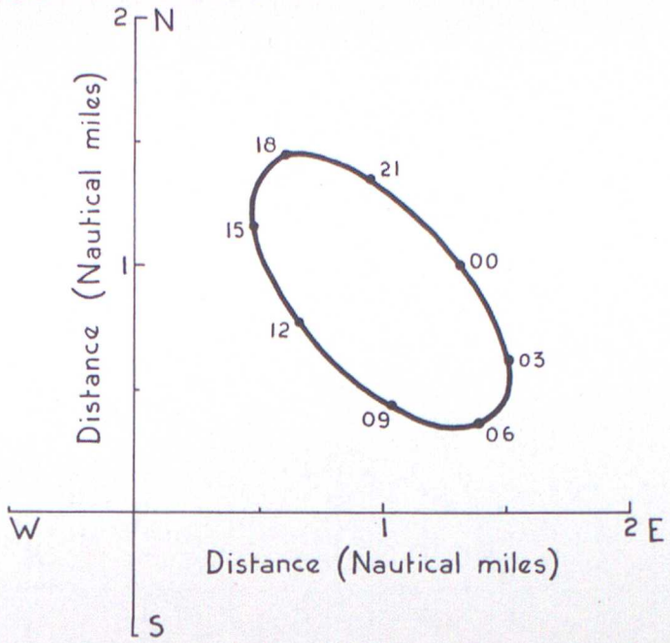


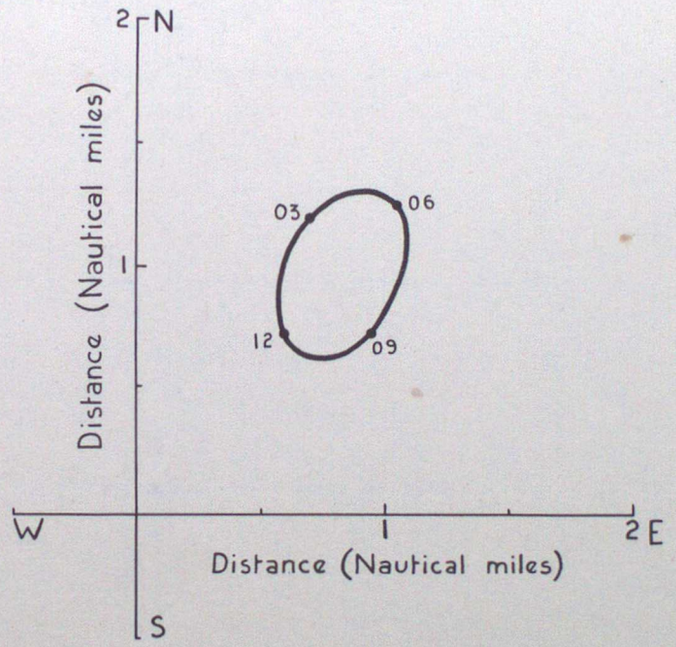
Fig. 5 The combined diurnal and semi-diurnal oscillations

(a) and (b) The crosses represent the observed deviation (given in Table V) of the wind components from the mean. The curves give the sum of the first two harmonics of these observed variations

(a) Diurnal oscillations



(b) Semi-diurnal oscillations



(c) Combined diurnal and semi-diurnal oscillations

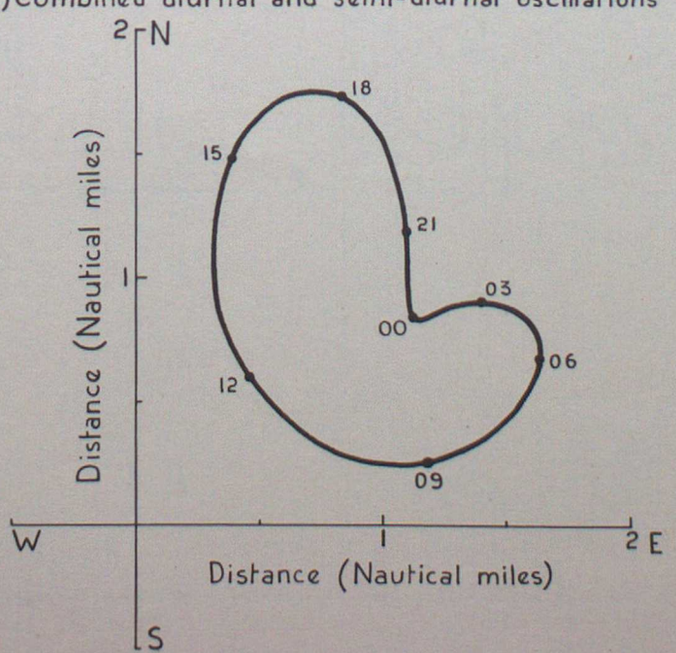


Fig. 6 Paths followed by particles oscillating under the wind systems defined by the diurnal and semi-diurnal variations of 100 mb. wind